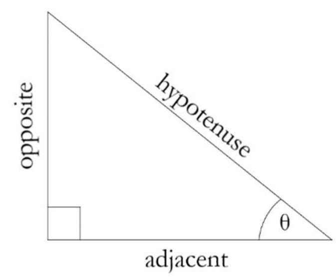
Trigonometric Identity I

$$\sin^2\theta + \cos^2\theta \equiv 1$$

This is a helpful equation used to relate the functions sine (otherwise known as sin) and cosine (otherwise known as cos). $\sin^2\theta$ is the same thing as $(\sin\theta)^2$, it is merely an easier way of writing it, the same is true for $\cos^2\theta$. The ≡ symbol means "identical to" (i.e. sine squared theta plus cosine squared theta is identical to 1). This symbols means the relationship is always true, regardless of the value of θ. θ is a placeholder for an angle, and for this identity to work the angle must be the same for both sine and cosine.

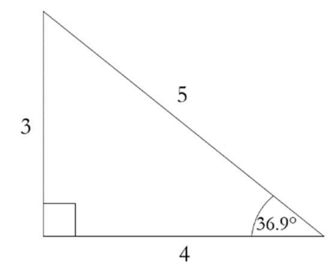


We know, from SOH CAH TOA, that for a triangle $\sin\theta = \frac{opposite}{hypotenuse}$ and $\cos\theta = \frac{adjacent}{hypotenus}$.

Therefore, using the example to the right, $\sin 36.9° = \frac{3}{5}$ and $\cos 36.9° = \frac{4}{5}$

N.B. 36.9° is a rounded value, the real value is 36.869897…°

$$\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2 = \frac{9}{25} + \frac{16}{25} = \frac{25}{25} = 1$$



Proof

We also know that, from Pythagoras' Theorem, that $hypotenuse^2 = opposite^2 + adjacent^2$

If we square $\sin\theta$ and $\cos\theta$ we get

$$\sin^2\theta = \frac{opposite^2}{hypotenuse^2} \text{ and } \cos^2\theta = \frac{adjacent^2}{hypotenuse^2}$$

Using SOH CAH TOA

$$\sin^2\theta + \cos^2\theta = \frac{opposit^{\ 2}}{hypotenus^{\ 2}} + \frac{adjacen^{\ 2}}{hypotenus^{\ 2}}$$

Adding these fractions together

$$\frac{opposite^2 + adjacent^2}{hypotenus^{\ 2}}$$

Using Pythagoras' Theorem

$$\frac{hypotenus^{\ 2}}{hypotenuse^2}$$

Any value (except 0) divided by itself is 1.

Therefore, we know that $\sin^2\theta + \cos^2\theta \equiv 1$

The hypotenuse cannot be of length 0 of course, otherwise it (and the triangle) would not exist.

See also
- Sine, Cosine and Tangent (SOH CAH TOA)
- Pythagoras' Theorem

References
Attwood, G. et al. (2017). *Edexcel AS and A level Mathematics - Pure - Year 1*. London: Pearson Education. p.209